

A NOVEL CONCEPT OF A MULTIPHASE, MULTI-MOTOR VECTOR CONTROLLED DRIVE SYSTEM SUPPLIED FROM A SINGLE VOLTAGE SOURCE INVERTER

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ABSTRACT: Since variable speed electric drive systems are supplied from power electronic converters, it is possible to utilise AC machines with a phase number higher than three. It is shown in the paper, using general theory of electrical machines, that an increase of the stator phase number to at least five (or more) enables completely independent vector control of two (or more) multiphase machines that are supplied from a single current-controlled voltage source inverter. In order to achieve such an independent control it is necessary to connect multiphase stator windings of the machines in series and perform an appropriate phase sequence transposition. The concept is equally applicable to any multiphase AC machine type and its major advantage, compared to an equivalent multi-motor three-phase drive system, is the saving of a certain number of inverter legs. The actual saving depends on the number of phases. The concept is introduced using an n -phase induction machine as the starting point and further analysis is restricted to an odd number of phases, for the reason explained in the paper. Classification of all the possible cases that may arise is given, followed by presentation of connection diagrams for selected phase numbers. Detailed verification of the proposed concept is provided by simulating the operation of a seven-phase three-motor drive system, controlled using indirect rotor flux oriented control principles. Some preliminary experimental results, which confirm the feasibility of a two-motor series-connected drive system, are included as well. The main advantages and drawbacks of the concept, when compared with an equivalent three-phase multi-motor drive system, are finally addressed.

KEYWORDS: Multi-motor drives, Multi-phase machines, Series connection, Vector control.

I. INTRODUCTION

Numerous applications, such as in textile manufacturing, HVAC, industrial manipulators, winders and electric vehicles, require more than one variable-speed electric drive. In the existing solutions a multi-drive system is realised with a common DC link, while each AC machine has its own voltage source inverter (VSI) as the supply. Machines and inverters used in these multi-drive systems are nowadays three-phase. Each machine can be controlled independently from the other machines, using its own voltage source inverter (VSI) and an appropriate control algorithm. A method that would enable completely independent control of at least two AC machines of different ratings and under different speed and loading conditions, while using only one VSI, does not exist at present. Such an algorithm is not possible in the three-phase case. The existing attempts to utilise a single three-phase inverter for supply and vector control of two or more three-phase machines connected in parallel are restricted to situations where speeds and loading of the machines are supposed to be the same [1-3].

In traditional electric machine applications a three-phase stator winding is selected, since the three-phase supply is readily available. However, when an AC machine is supplied from an inverter, the need for a pre-defined number of phases on stator, such as three, disappears and other phase numbers can be chosen. Probably the first proposal of a multiphase variable speed electric drive dates back to 1969 [4]. While [4] proposed a five-phase induction motor drive, a six-phase (double star) induction machine supplied from a six-phase inverter was examined in [5,6]. The early interest in multiphase machines was caused by the possibility of reducing the torque ripple in inverter fed drives, when compared to the three-phase case. Another advantage of a multiphase motor drive over a three-phase motor drive is an improved reliability due to fault tolerance features [7-10], this being one of the main reasons behind the application of six-phase (double-star) and nine-phase (triple-star) induction motor drives in locomotives [7,8]. The other main reason is that for a given motor power an increase of the number of phases enables reduction of the power per phase, which translates into a reduction of the power per inverter leg (that is, a semiconductor rating). Multiphase machines are therefore often considered for and applied in high power applications [11,12]. Other advantages of multiphase machines over their three-phase counterparts include an improvement in the noise characteristics [13] and a possibility of reduction in the stator copper loss, leading to an improvement in the efficiency [14]. Recent surveys of the state-of-the art in this area [15,16] indicate an ever increasing interest in multiphase machines within the scientific community world-wide.

Vector control principles can be extended from a three-phase to a multiphase machine in a simple manner when the machine torque is produced by the fundamental stator current component only. For example, a vector control scheme for a five-phase synchronous reluctance machine is detailed in [17], while vector control of a five-phase induction machine is elaborated in [18]. In principle, there is not any difference with regard to the vector control scheme between a three-phase and an n -phase machine.

The purpose of this paper is to develop a concept for a multiphase multi-motor vector controlled drive system in which stator windings of the machines are connected in series, with an appropriate phase transposition, and the supply is a single current-controlled VSI. Such a connection, as shown in the paper, enables completely independent vector control of the given number of machines, meaning that there are no restrictions whatsoever on the power rating, speeds and loading of the individual machines (in practice, the machines' power rating will have to be matched to avoid excessive additional loss production caused by the series connection). It should be noted that the machine type is irrelevant in the context of this paper, since the basic idea applies equally to

multiphase induction and synchronous motors (all versions). The only requirement is that the supply can be treated as an equivalent current source, meaning that the VSI is current-controlled, since vector control with current control in the stationary reference frame is applied. The idea of such a drive system was for the first time proposed in [19], where the concept of an n -dimensional space for an n -phase machine [20,21] was applied in the analysis. The concept is developed here using general theory of electric machines [22,23] in a systematic way. It is further extended to an arbitrary odd number of phases, and a classification of various cases that may occur, depending on the number of phases, is provided. Connection diagrams and so-called ‘connectivity matrices’ are introduced for selected phase numbers. The proposed concept of a multiphase multi-motor drive system is finally verified, at first by performing detailed simulation studies for a vector controlled seven-phase three-motor system in both torque and speed mode of operation. Next, the design of the experimental rig is briefly described and preliminary experimental results for a two-motor series-connected drive are presented. The main advantages and shortcomings of the drive system are highlighted.

The paper is organised as follows. Section II reviews the models of a generalised n -phase induction machine and introduces the basic idea of the concept. An explanation why the analysis is restricted to odd numbers of phases is provided in this section as well. Section III gives guidelines for constructing the connection diagrams and connectivity matrices for various selected phase number cases. Section IV performs a classification of all the possible situations that may arise for an odd number of phases. Section V briefly reviews the indirect vector control algorithm as applicable to a seven-phase induction machine and explains correlation between source voltages and currents and individual voltages and flux/torque producing currents of the machines connected in series. Simulation results for a seven-phase three-motor drive system, for torque mode and speed mode of operation, as well as experimental results for a two-motor drive system, are given in Section VI. Section VII briefly discusses major advantages and shortcoming of the proposed concept, while the conclusions of the study are summarised in Section VIII.

II. MODELLING OF AN n -PHASE INDUCTION MACHINE

A. Phase-variable model

An n -phase induction machine, such that the spatial displacement between any two mmfs produced by any two consecutive phases is $\alpha = 2\pi/n$, is under consideration. For the sake of generality both stator and rotor are assumed to be of n -phase structure. All the standard assumptions of the general theory of electrical machines apply. Stator and rotor voltage equilibrium equations and flux linkage equations can then be written as:

$$\underline{v}^s = \underline{R}_s \underline{i}^s + \frac{d\underline{\psi}^s}{dt} \quad \underline{\psi}^s = \underline{L}_s \underline{i}^s + \underline{L}_{sr} \underline{i}^r \quad (1)$$

$$\underline{v}^r = \underline{R}_r \underline{i}^r + \frac{d\underline{\psi}^r}{dt} \quad \underline{\psi}^r = \underline{L}_r \underline{i}^r + \underline{L}_{rs} \underline{i}^s \quad (2)$$

Let the phases of both stator and rotor be identified with numbers $1,2,3,\dots,n$ according to the spatial ordering of the windings along the circumference of the stator and rotor. Spatial displacement between any two consecutive windings is therefore $\alpha = 2\pi/n$. The following definition of phase voltages, currents and flux linkages applies to (1)-(2):

$$\begin{aligned}\underline{v}^s &= [v_{1s} \ v_{2s} \ v_{3s} \ \dots \ v_{ns}]^T \\ \underline{i}^s &= [i_{1s} \ i_{2s} \ i_{3s} \ \dots \ i_{ns}]^T \\ \underline{\psi}^s &= [\psi_{1s} \ \psi_{2s} \ \psi_{3s} \ \dots \ \psi_{ns}]^T\end{aligned}\quad (3)$$

$$\begin{aligned}\underline{v}^r &= [v_{1r} \ v_{2r} \ v_{3r} \ \dots \ v_{nr}]^T \\ \underline{i}^r &= [i_{1r} \ i_{2r} \ i_{3r} \ \dots \ i_{nr}]^T \\ \underline{\psi}^r &= [\psi_{1r} \ \psi_{2r} \ \psi_{3r} \ \dots \ \psi_{nr}]^T\end{aligned}\quad (4)$$

It is assumed that the rotor winding has already been referred to stator winding. The stator and rotor inductance matrices are given with ($\alpha = 2\pi/n$):

$$\underline{L}_s = \begin{bmatrix} L_{11s} & L_{12s} & L_{13s} & \dots & L_{1ns} \\ L_{21s} & L_{22s} & L_{23s} & \dots & L_{2ns} \\ L_{31s} & L_{32s} & L_{33s} & \dots & L_{3ns} \\ \dots & \dots & \dots & \dots & \dots \\ L_{n1s} & L_{n2s} & L_{n3s} & \dots & L_{nns} \end{bmatrix} = \begin{bmatrix} L_{1s} + M & M \cos \alpha & M \cos 2\alpha & \dots & M \cos(n-1)\alpha \\ M \cos(n-1)\alpha & L_{1s} + M & M \cos \alpha & \dots & M \cos(n-2)\alpha \\ M \cos(n-2)\alpha & M \cos(n-1)\alpha & L_{1s} + M & \dots & M \cos(n-3)\alpha \\ \dots & \dots & \dots & \dots & \dots \\ M \cos \alpha & M \cos 2\alpha & M \cos 3\alpha & \dots & L_{1s} + M \end{bmatrix} \quad (5)$$

$$\underline{L}_r = \begin{bmatrix} L_{11r} & L_{12r} & L_{13r} & \dots & L_{1nr} \\ L_{21r} & L_{22r} & L_{23r} & \dots & L_{2nr} \\ L_{31r} & L_{32r} & L_{33r} & \dots & L_{3nr} \\ \dots & \dots & \dots & \dots & \dots \\ L_{n1r} & L_{n2r} & L_{n3r} & \dots & L_{nnr} \end{bmatrix} = \begin{bmatrix} L_{1r} + M & M \cos \alpha & M \cos 2\alpha & \dots & M \cos(n-1)\alpha \\ M \cos(n-1)\alpha & L_{1r} + M & M \cos \alpha & \dots & M \cos(n-2)\alpha \\ M \cos(n-2)\alpha & M \cos(n-1)\alpha & L_{1r} + M & \dots & M \cos(n-3)\alpha \\ \dots & \dots & \dots & \dots & \dots \\ M \cos \alpha & M \cos 2\alpha & M \cos 3\alpha & \dots & L_{1r} + M \end{bmatrix} \quad (6)$$

while the stator-to-rotor mutual inductance matrices are

$$\underline{L}_{sr} = M \begin{bmatrix} \cos \theta & \cos(\theta + \alpha) & \cos(\theta + 2\alpha) & \dots & \cos(\theta + (n-1)\alpha) \\ \cos(\theta + (n-1)\alpha) & \cos \theta & \cos(\theta + \alpha) & \dots & \cos(\theta + (n-2)\alpha) \\ \cos(\theta + (n-2)\alpha) & \cos(\theta + (n-1)\alpha) & \cos \theta & \dots & \cos(\theta + (n-3)\alpha) \\ \dots & \dots & \dots & \dots & \dots \\ \cos(\theta + \alpha) & \cos(\theta + 2\alpha) & \cos(\theta + 3\alpha) & \dots & \cos \theta \end{bmatrix} \quad (7)$$

$$\underline{L}_{rs} = \underline{L}_{sr}^T$$

The angle θ denotes the instantaneous position of the magnetic axis of the rotor phase 'a' with respect to the stationary magnetic axis of the stator phase 'a' (i.e. the instantaneous position of the rotor with respect to the stator). Stator and rotor resistance matrices are diagonal $n \times n$ matrices,

$$\begin{aligned}\underline{R}_s &= \text{diag}(R_s \ R_s \ R_s \ \dots \ R_s) \\ \underline{R}_r &= \text{diag}(R_r \ R_r \ R_r \ \dots \ R_r)\end{aligned}\quad (8)$$

Motor torque can be expressed in terms of stator and rotor phase currents as

$$T_e = -PM \left\{ \begin{aligned} & (i_{1s}i_{1r} + i_{2s}i_{2r} + i_{3s}i_{3r} + \dots + i_{ns}i_{nr}) \sin \theta + (i_{ns}i_{1r} + i_{1s}i_{2r} + i_{2s}i_{3r} + i_{3s}i_{4r} + \dots + i_{(n-1)s}i_{nr}) \sin(\theta + \alpha) + \\ & (i_{(n-1)s}i_{1r} + i_{ns}i_{2r} + i_{1s}i_{3r} + i_{2s}i_{4r} + \dots + i_{(n-2)s}i_{nr}) \sin(\theta + 2\alpha) + \\ & (i_{(n-2)s}i_{1r} + i_{(n-1)s}i_{2r} + i_{ns}i_{3r} + i_{1s}i_{4r} + \dots + i_{(n-3)s}i_{nr}) \sin(\theta + 3\alpha) + \\ & \dots + \\ & (i_{2s}i_{1r} + i_{3s}i_{2r} + i_{4s}i_{3r} + i_{5s}i_{4r} + \dots + i_{1s}i_{nr}) \sin(\theta + (n-1)\alpha) \end{aligned} \right\} \quad (9)$$

B. Application of the decoupling transformation

Let the transformation be defined with the following correlations

$$\begin{aligned} \underline{v}_{\alpha\beta}^s &= \underline{C} \underline{v}^s & \underline{i}_{\alpha\beta}^s &= \underline{C} \underline{i}^s & \underline{\psi}_{\alpha\beta}^s &= \underline{C} \underline{\psi}^s \\ \underline{v}_{\alpha\beta}^r &= \underline{C} \underline{v}^r & \underline{i}_{\alpha\beta}^r &= \underline{C} \underline{i}^r & \underline{\psi}_{\alpha\beta}^r &= \underline{C} \underline{\psi}^r \end{aligned} \quad (10)$$

where the decoupling transformation matrix is, for an odd number of phases and using power invariant from, given with [23]:

$$\underline{C} = \sqrt{\frac{2}{n}} \begin{bmatrix} 1 & \cos \alpha & \cos 2\alpha & \cos 3\alpha & \dots & \cos 3\alpha & \cos 2\alpha & \cos \alpha \\ 0 & \sin \alpha & \sin 2\alpha & \sin 3\alpha & \dots & -\sin 3\alpha & -\sin 2\alpha & -\sin \alpha \\ 1 & \cos 2\alpha & \cos 4\alpha & \cos 6\alpha & \dots & \cos 6\alpha & \cos 4\alpha & \cos 2\alpha \\ 0 & \sin 2\alpha & \sin 4\alpha & \sin 6\alpha & \dots & -\sin 6\alpha & -\sin 4\alpha & -\sin 2\alpha \\ 1 & \cos 3\alpha & \cos 6\alpha & \cos 9\alpha & \dots & \cos 9\alpha & \cos 6\alpha & \cos 3\alpha \\ 0 & \sin 3\alpha & \sin 6\alpha & \sin 9\alpha & \dots & -\sin 9\alpha & -\sin 6\alpha & -\sin 3\alpha \\ 1 & \cos 4\alpha & \cos 8\alpha & \cos 12\alpha & \dots & \cos 12\alpha & \cos 8\alpha & \cos 4\alpha \\ 0 & \sin 4\alpha & \sin 8\alpha & \sin 12\alpha & \dots & -\sin 12\alpha & -\sin 8\alpha & -\sin 4\alpha \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & \cos\left(\frac{n-1}{2}\alpha\right) & \cos 2\left(\frac{n-1}{2}\alpha\right) & \cos 3\left(\frac{n-1}{2}\alpha\right) & \dots & \cos 3\left(\frac{n-1}{2}\alpha\right) & \cos 2\left(\frac{n-1}{2}\alpha\right) & \cos\left(\frac{n-1}{2}\alpha\right) \\ 0 & \sin\left(\frac{n-1}{2}\alpha\right) & \sin 2\left(\frac{n-1}{2}\alpha\right) & \sin 3\left(\frac{n-1}{2}\alpha\right) & \dots & -\sin 3\left(\frac{n-1}{2}\alpha\right) & -\sin 2\left(\frac{n-1}{2}\alpha\right) & -\sin\left(\frac{n-1}{2}\alpha\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \dots & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (11)$$

Transformation of the model (1)-(9) using (11) yields the following stator voltage equations ($L_m = (n/2)M$):

$$\begin{aligned} v_{\alpha s} &= R_s i_{\alpha s} + \frac{d\psi_{\alpha s}}{dt} = R_s i_{\alpha s} + (L_{ls} + L_m) \frac{di_{\alpha s}}{dt} + L_m \frac{d}{dt} (i_{\alpha r} \cos \theta - i_{\beta r} \sin \theta) \\ v_{\beta s} &= R_s i_{\beta s} + \frac{d\psi_{\beta s}}{dt} = R_s i_{\beta s} + (L_{ls} + L_m) \frac{di_{\beta s}}{dt} + L_m \frac{d}{dt} (i_{\alpha r} \sin \theta + i_{\beta r} \cos \theta) \\ v_{x1s} &= R_s i_{x1s} + \frac{d\psi_{x1s}}{dt} = R_s i_{x1s} + L_{ls} \frac{di_{x1s}}{dt} \\ v_{y1s} &= R_s i_{y1s} + \frac{d\psi_{y1s}}{dt} = R_s i_{y1s} + L_{ls} \frac{di_{y1s}}{dt} \\ \dots & \\ \dots & \\ v_{0s} &= R_s i_{0s} + \frac{d\psi_{0s}}{dt} = R_s i_{0s} + L_{ls} \frac{di_{0s}}{dt} \end{aligned} \quad (12)$$

Rotor voltage equations become

$$\begin{aligned}
v_{\alpha r} = 0 &= R_r i_{\alpha r} + \frac{d\psi_{\alpha r}}{dt} = R_r i_{\alpha r} + (L_{lr} + L_m) \frac{di_{\alpha r}}{dt} + L_m \frac{d}{dt} (i_{\alpha s} \cos \theta + i_{\beta s} \sin \theta) \\
v_{\beta r} = 0 &= R_r i_{\beta r} + \frac{d\psi_{\beta r}}{dt} = R_r i_{\beta r} + (L_{lr} + L_m) \frac{di_{\beta r}}{dt} + L_m \frac{d}{dt} (-i_{\alpha s} \sin \theta + i_{\beta s} \cos \theta) \\
v_{x1r} = 0 &= R_r i_{x1r} + \frac{d\psi_{x1r}}{dt} = R_r i_{x1r} + L_{lr} \frac{di_{x1r}}{dt} \\
v_{y1r} = 0 &= R_r i_{y1r} + \frac{d\psi_{y1r}}{dt} = R_r i_{y1r} + L_{lr} \frac{di_{y1r}}{dt} \\
\text{.....} & \\
\text{.....} & \\
v_{0r} = 0 &= R_r i_{0r} + \frac{d\psi_{0r}}{dt} = R_r i_{0r} + L_{lr} \frac{di_{0r}}{dt}
\end{aligned} \tag{13}$$

while the machine's torque can be expressed as

$$T_e = PL_m \left[\cos \theta (i_{\alpha r} i_{\beta s} - i_{\beta r} i_{\alpha s}) - \sin \theta (i_{\alpha r} i_{\alpha s} + i_{\beta r} i_{\beta s}) \right] \tag{14}$$

In the case of an even number of phases the decoupling transformation matrix contains two identical rows like the last one in (11), yielding in the general case two zero sequence components. Hence in (12)-(13) the last two voltage equations are for zero sequence components for an even number of phases.

Torque equation (14) shows that the machine's torque is entirely developed due to the interaction of stator/rotor α - β current components and is independent of the value of x-y current components, regardless of the number of phases. Hence, for any n -phase machine flux and torque control require only two current components. From rotor equations (13) it follows that, since the rotor is short-circuited and stator x-y components are decoupled from rotor x-y components, all rotor x-y components and zero sequence components are identically equal to zero. The same applies to the stator zero sequence components, since star connection is assumed.

It follows from (12)-(13) that for an n -phase machine there are $(n-1)/2$ pairs of current components (one α - β pair and $[(n-1)/2-1]$ x-y pairs of components) when the number of phases is an odd number. If the number of phases is an even number there are $(n-2)/2$ pairs of current components (one α - β pair and $[(n-2)/2-1]$ x-y pairs of components). As one pair of current components is required for control of one machine, then the number of available pairs of current components (i.e. the number of machines that could be controlled independently) is the same for an odd phase number n and the subsequent even phase number $n+1$. The even number asks for one more inverter leg than the previous odd number, while offering no obvious advantages that would compensate for this shortcoming. It is for this reason that the discussion is further on restricted to the odd phase numbers. Nevertheless, for even phase numbers greater than or equal to eight, there appears a saving in the number of inverter legs as well, when compared to an equivalent three-phase multi-motor drive system. A general study for all the possible even phase numbers has also been conducted and the findings are reported in [24].

C. Rotational transformation

Since all the x-y components and zero sequence components of stator and rotor are decoupled, rotational transformation needs to be applied to α - β components only. The transformation matrices for stator and rotor are:

$$\begin{aligned}
\underline{D}_s &= \begin{bmatrix} \cos \theta_s & \sin \theta_s & & & \\ -\sin \theta_s & \cos \theta_s & & & \\ & & 1 & & \\ & & & \dots & \\ & & & & 1 \end{bmatrix} & \underline{D}_s^{-1} &= \begin{bmatrix} \cos \theta_s & -\sin \theta_s & & & \\ \sin \theta_s & \cos \theta_s & & & \\ & & 1 & & \\ & & & \dots & \\ & & & & 1 \end{bmatrix} \\
\underline{D}_r &= \begin{bmatrix} \cos \beta & \sin \beta & & & \\ -\sin \beta & \cos \beta & & & \\ & & 1 & & \\ & & & \dots & \\ & & & & 1 \end{bmatrix} & \underline{D}_r^{-1} &= \begin{bmatrix} \cos \beta & -\sin \beta & & & \\ \sin \beta & \cos \beta & & & \\ & & 1 & & \\ & & & \dots & \\ & & & & 1 \end{bmatrix}
\end{aligned} \tag{15}$$

Here θ_s is the instantaneous angular position of the d-axis of the common reference frame with respect to the phase 'a' magnetic axis of the stator, while $\beta = \theta_s - \theta$ is the instantaneous angular position of the d-axis of the common reference frame with respect to the phase 'a' magnetic axis of the rotor. A general case of an arbitrary reference frame is considered, so that $\theta_s = \int \omega_a dt$.

Upon application of (15) in conjunction with the model (12)-(14), stator and rotor voltage equations take the form

$$\begin{aligned}
v_{ds} &= R_s i_{ds} - \omega_a \psi_{qs} + p \psi_{ds} & v_{dr} &= 0 = R_r i_{dr} - (\omega_a - \omega) \psi_{qr} + p \psi_{dr} \\
v_{qs} &= R_s i_{qs} + \omega_a \psi_{ds} + p \psi_{qs} & v_{qr} &= 0 = R_r i_{qr} + (\omega_a - \omega) \psi_{dr} + p \psi_{qr} \\
v_{x1s} &= R_s i_{x1s} + p \psi_{x1s} & v_{x1r} &= 0 = R_r i_{x1r} + p \psi_{x1r} \\
v_{y1s} &= R_s i_{y1s} + p \psi_{y1s} & v_{y1r} &= 0 = R_r i_{y1r} + p \psi_{y1r}
\end{aligned} \tag{16}$$

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$$\begin{aligned}
v_{os} &= R_s i_{os} + p \psi_{os} & v_{or} &= R_r i_{or} + p \psi_{or}
\end{aligned}$$

where $p \equiv d/dt$. Flux linkages are given with

$$\begin{aligned}
\psi_{ds} &= (L_{ls} + L_m) i_{ds} + L_m i_{dr} & \psi_{dr} &= (L_{lr} + L_m) i_{dr} + L_m i_{ds} \\
\psi_{qs} &= (L_{ls} + L_m) i_{qs} + L_m i_{qr} & \psi_{qr} &= (L_{lr} + L_m) i_{qr} + L_m i_{qs} \\
\psi_{x1s} &= L_{ls} i_{x1s} & \psi_{x1r} &= L_{lr} i_{x1r} \\
\psi_{y1s} &= L_{ls} i_{y1s} & \psi_{y1r} &= L_{lr} i_{y1r}
\end{aligned} \tag{17}$$

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$$\begin{aligned}
\psi_{os} &= L_{ls} i_{os} & \psi_{or} &= L_{lr} i_{or}
\end{aligned}$$

Here once more $L_m = (n/2)M$. Torque equation is

$$T_e = PL_m [i_{dr} i_{qs} - i_{ds} i_{qr}] \tag{18}$$

Machine model described with (16)-(18) is identical as for a three-phase machine, except for the presence of x-y component equations. As rotor flux and torque are independent of x-y components, vector control scheme for an n-phase induction machine is the same as for a three-phase machine, except that coordinate transformation involves creation of n individual phase currents (rather than three) from the d-q axis stator current references.

D. *The basic idea of the multiphase series-connected multi-motor system*

Since only one pair of stator α - β current components is needed for the flux and torque control in one machine, there is a possibility of using the existing degrees of freedom (i.e. $[(n-1)/2 - 1]$ x-y pairs of stator current components) for control of other machines that would be connected in series with the first machine. However, if the control of the machines with series connected stator windings is to be decoupled one from the other, it is necessary that the flux/torque producing currents of one machine do not produce flux and torque in all the other machines in the group. In other words, if it is possible to connect stator windings of $m = (n-1)/2$ multiphase machines in such a way that what one machine sees as the d-q axis stator current components the other machines see as x-y current components, and vice versa, it would become possible to completely independently control speed (position, torque) of these $(n-1)/2$ machines while supplying the machines from a single current-controlled voltage source inverter. In simple terms, and taking the five-phase machine case, it will become possible to independently realise vector control of two five phase machines using a single voltage source inverter, provided that the stator windings of the two machines are connected in series and that an appropriate phase transposition is introduced so that the set of five five-phase currents that produce rotating mmf in the first machine, does not produce rotating mmf in the second machine and vice versa. This explanation constitutes the basis of the multi-motor multiphase drive system that is to be described further on. In the general case a set of n -phase stator windings is to be supplied from a single n -phase current-controlled (CC) voltage source by connecting the stator windings in series and using the phase transposition, as illustrated in Fig. 1 (source phases are identified with capital letters A,B,C...). Phase transposition means shift in connection of the phases 1,2, ... n of one machine to the phases 1,2, ... n of the second machine, etc., where 1,2,3... n is the flux/torque producing phase sequence of the given machine according to the spatial distribution of the phases within the stator winding. The issue of required phase transposition is discussed in detail in the next Section.

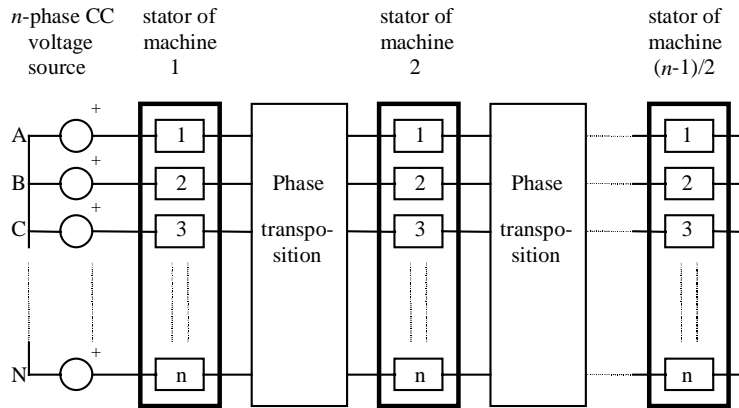


Fig. 1. Supply of $(n-1)/2$ machine stator windings, connected in series, from an n -phase current-controlled voltage source.

III. CONNECTION DIAGRAMS AND CONNECTIVITY MATRICES

A. *General n -phase case*

If a set of n -phase machines is to be connected in series and the machines are to be controlled independently, it is necessary to make flux/torque producing currents of one machine non flux/torque producing currents in all

the other machines. In other words α - β currents of one machine must appear as one of the x-y pairs of currents for all the other machines. Simple series connection of stator windings will of course not yield the desired result. However, if phase transposition is introduced between phases in series connection, the desired effect will automatically result. An answer to the question how the phase transposition needs to be done lies in the decoupling transformation matrix (11).

According to the transformation matrix (11), phases '1' of all the machines will be connected directly in series (the first column in (11)). The phase transposition for phase '1' is therefore 0 degrees and the phase step is zero. However, phase '2' of the first machine will be connected to phase '3' of the second machine, which will be further connected to phase '4' of the third machine and so on. The phase transposition moving from one machine to the other is the spatial angle α and the phase step is 1. This follows from the second column of the transformation matrix that contains cosine and sine terms with spatial displacements equal to $\alpha, 2\alpha, 3\alpha, 4\alpha, 5\alpha, \dots [(n-1)/2]\alpha$. In a similar manner phase '3' of the first machine (third element in the first row of (11) with spatial displacement of 2α) is connected to the phase '5' of the second machine, which further gets connected to phase '7' of the third machine, and so on. The phase transposition is 2α , and the phase step is 2. This follows from the third column of the transformation matrix, where spatial displacement equals $2\alpha, 4\alpha, 6\alpha, \dots 2[(n-1)/2]\alpha$. Further, phase '4' of the first machine needs to be connected to the phase '7' of the second machine which gets connected to the phase '10' of the third machine and so on. Here the phase step is equal to 3 and the phases are transposed by 3α . This corresponds to the fourth column in the transformation matrix, where terms with $3\alpha, 6\alpha, 9\alpha, 12\alpha, \dots 3[(n-1)/2]\alpha$ appear. For phase '5' of the first machine the phase transposition will equal 4α and phase step will be 4, for phase '6' the phase step will be 5 and phase transposition will equal 5α , and so on.

The above given explanation enables construction of a connection table, which is called further on connectivity matrix. In the general case of an n -phase system, connectivity matrix of the form given in Table I results. Flux/torque producing phase sequence for any particular machine is denoted in Table I with symbols a,b,c,d,... rather than with numbers 1,2,3,... (both notations are used further on, depending on which one is more convenient for the given purpose).

Table I: Connectivity matrix for the general n -phase case.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
M1	a	b	c	d	e	f	g	h	I	j	k	l	m	n	...
M2	a	b+1	c+2	d+3	e+4	f+5	g+6	h+7	i+8	j+9	k+10	l+11	m+12	n+13	...
M3	a	b+2	c+4	d+6	e+8	f+10	g+12	h+14	i+16	j+18	k+20	l+22	m+24	n+26	...
M4	a	b+3	c+6	d+9	e+12	f+15	g+18	h+21	i+24	j+27	k+30	l+33	m+36	n+39	...
M5	a	b+4	c+8	d+12	e+16	f+20	g+24	h+28	i+32	j+36	k+40	l+44	m+48	n+52	...
M6	a	b+5	c+10	d+15	e+20	f+25	g+30	h+35	i+40	j+45	k+50	l+55	m+60	n+65	...
M7	a	b+6	c+12	d+18	e+24	f+30	g+36	h+42	i+48	j+54	k+60	l+66	m+72	n+78	...
....

Double-line in Table I encircles the seven-phase case, while the bold box applies to the five-phase case. Dashed line encircles eleven-phase case, while solid line is the box for the thirteen-phase case. If a number in the table, obtained by substituting $a=1, b=2, c=3, d=4$, etc., is greater than the number of phases n , resetting is performed by deducting $j \times n$ ($j = 1,2,3,\dots$) from the number so that the resulting number belongs to the set $[1,n]$.

Connection rules, described in this sub-section and summarised in the general connectivity matrix in Table I, can be expressed in a form of symbolic equations. Let the machines be denoted as M_1, M_2, \dots, M_k , where k stands for the maximum number of connectable machines (equal to $(n - 1)/2$). Similarly, let the machine phases be denoted for each machine, according to the sequence of spatial ordering, with indices $1, 2, 3, \dots, n$ and let the source phases be $SP_1, SP_2, SP_3, \dots, SP_n$ (instead of A,B,C,D...). The connection of the appropriate machine phases to individual source phases then proceeds in accordance with the following symbolic equations:

$$\begin{aligned}
 SP_1 &= M1_1 \rightarrow M2_1 \rightarrow M3_1 \rightarrow M4_1 \rightarrow \dots \rightarrow M_k_1 \\
 SP_2 &= M1_2 \rightarrow M2_{2+1} \rightarrow M3_{2+2} \rightarrow M4_{2+3} \rightarrow \dots \rightarrow M_k_{2+(k-1)} \\
 SP_3 &= M1_3 \rightarrow M2_{3+2} \rightarrow M3_{3+4} \rightarrow M4_{3+6} \rightarrow \dots \rightarrow M_k_{3+2(k-1)} \\
 SP_4 &= M1_4 \rightarrow M2_{4+3} \rightarrow M3_{4+6} \rightarrow M4_{4+9} \rightarrow \dots \rightarrow M_k_{4+3(k-1)} \\
 &\text{-----} \\
 SP_n &= M1_n \rightarrow M2_{n+(n-1)} \rightarrow M3_{n+2(n-1)} \rightarrow M4_{n+3(n-1)} \rightarrow \dots \rightarrow M_k_{n+(n-1)(k-1)}
 \end{aligned} \tag{19}$$

Note that the meaning of the equality sign in (19) is ‘source phase consists of’, while the meaning of the symbol \rightarrow is ‘connect to’. On the basis of (19) it is possible to write a symbolic equation for the connection rule for any of the source phases $j = 1, 2, 3, \dots, n$ in the following form:

$$SP_j = M1_j \rightarrow M2_{j+(j-1)} \rightarrow M3_{j+2(j-1)} \rightarrow M4_{j+3(j-1)} \rightarrow \dots \rightarrow M_k_{j+(j-1)(k-1)} \tag{20}$$

Symbolic equation (20) can be finally given in a compact form as

$$SP_j = \sum_{i=1}^k Mi_{j+(i-1)(j-1)} \quad j = 1, 2, 3, \dots, n \tag{21}$$

where the meaning of the summation symbol is again symbolic.

B. Selected specific numbers of phases

Connectivity matrix and the corresponding connection diagrams are given in this sub-section for certain selected specific phase numbers. Selection is done in such a way as to aid the discussion of the classification of all the possible situations in the next Section.

Consider at first the five-phase case [19]. Table II and Fig. 2 give the connectivity matrix and the connection diagram for the two five-phase machines, which are obtained from the general case shown in Fig. 1, Table I and equation (21).

Table II: Connectivity matrix for the five-phase case.

	A	B	C	D	E
M1	1	2	3	4	5
M2	1	3	5	2	4

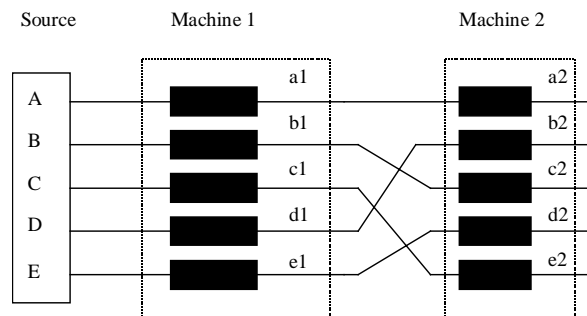


Fig. 2: Connection diagram for the five-phase case.

Situation that arises in the seven-phase case is shown in Table III and Fig. 3. Three seven-phase machines can be now connected in series, as follows from the discussion in sub-section II.D.

Table III: Connectivity matrix for the seven-phase case.

	A	B	C	D	E	F	G
M1	1	2	3	4	5	6	7
M2	1	3	5	7	2	4	6
M3	1	4	7	3	6	2	5

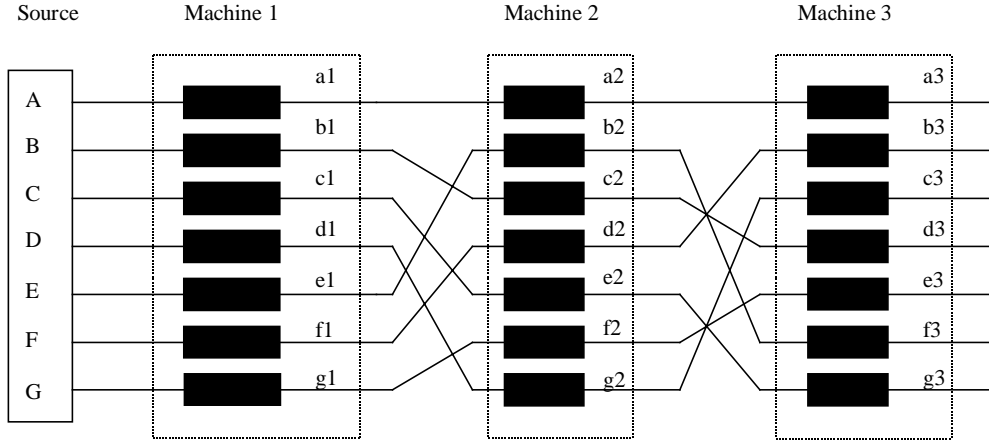


Fig. 3: Connection diagram for three seven-phase machines.

The two phase numbers considered so far are both prime numbers and the situation is simple. However, when the number of phases is not a prime number the situation becomes more involved. Consider the nine-phase case. Connectivity matrix, obtained from Table I, is shown in Table IV. As can be seen in Table IV, only phases 1,4 and 7 of machine M3 (encircled with a dotted box) are utilized in the series connection. Since the spatial displacement between these phases is 120 degrees, this means that machine M3 is actually a three-phase rather than a nine-phase machine. Connection diagram is shown in Fig. 4. It has to be noted that a re-ordering of the machines has to be done if series connection comprises machines of different phase numbers, as shown in Fig. 4. All the machines with the highest phase number have to be connected at first in series to the source, respecting the required phase transposition. Next come all the machines with the second highest phase number, and so on. The sequence finishes with the smallest phase number. This is so since, taking the nine-phase case as the example, the three flux/torque producing currents of a nine-phase machine which enter any given phase of the three-phase machine sum to zero in any instant in time. This simultaneously means that the three-phase machine in Fig. 4 will not suffer any adverse effects due to the series connection with three nine-phase machines.

Table IV: Connectivity matrix for the nine-phase case.

	A	B	C	D	E	F	G	H	I
M1	1	2	3	4	5	6	7	8	9
M2	1	3	5	7	9	2	4	6	8
M3	1	4	7	1	4	7	1	4	7
M4	1	5	9	4	8	3	7	2	6

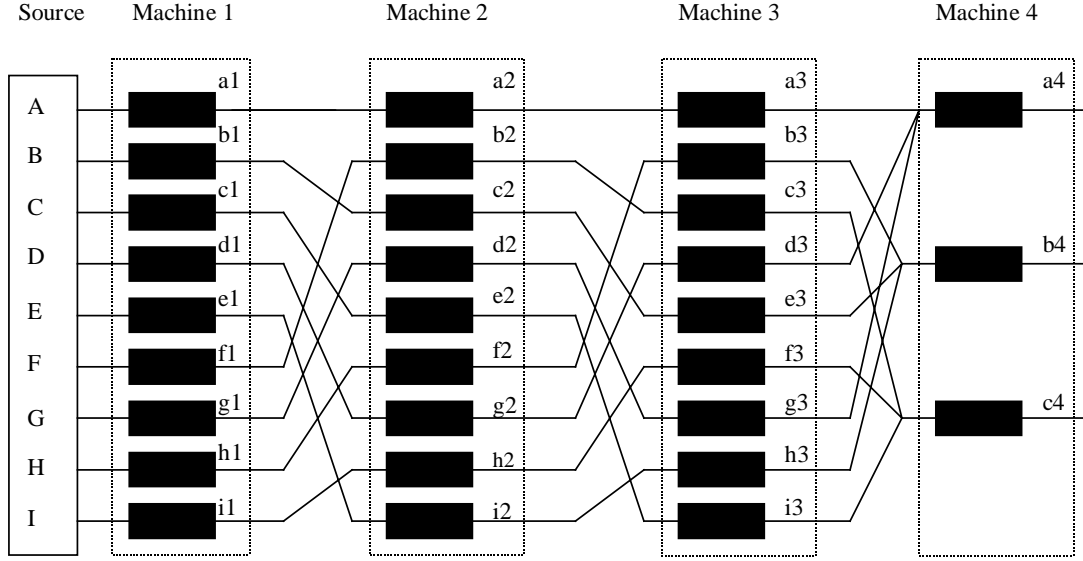


Fig. 4: Connection diagram for the nine-phase case: three nine-phase machines and one three-phase machine are connected in series.

The last case illustrated here is the fifteen-phase case. Table V gives the connectivity matrix. As can be seen, only terminals 1,4,7,10,13 are utilised by two of the seven machines that should be connectable (M3 and M6, encircled with dotted line). Since the spatial displacement between terminals for these two machines is 72 degrees, this implies that the two machines are five-phase. In addition to two five-phase machines, connectivity matrix contains however one three-phase machine as well, machine M5 (double-line box; terminals 1,6 and 11 are utilised only, with spatial displacement of 120 degrees, so that the machine is a three-phase one). Connectivity matrix therefore implies a possibility of connecting seven machines in series: four are fifteen-phase, two are five-phase and one is three-phase. However, it can be easily shown that a five-phase machine cannot be connected to a three-phase machine (such an attempt would lead to a complete short-circuiting of the terminals of the two machines, since all the five phases of a five-phase machine would need to be connected to all the three phases of the three-phase machine). Hence, in the fifteen-phase case, the total number of machines that can be connected in series is smaller than the one predicted from the analysis in Section II.D. Instead of seven machines, at most six machines can be connected (four fifteen-phase and two five-phase). Alternatively, it is possible to connect five machines altogether, by using the three-phase machine and four fifteen-phase machines.

The four specific phase number cases covered in this sub-section constitute the basis for classification of all possible situations in the next Section.

Table V: Connectivity matrix for the fifteen-phase case.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
M1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
M2	1	3	5	7	9	11	13	15	2	4	6	8	10	12	14
M3	1	4	7	10	13	1	4	7	10	13	1	4	7	10	13
M4	1	5	9	13	2	6	10	14	3	7	11	15	4	8	12
M5	1	6	11	1	6	11	1	6	11	1	6	11	1	6	11
M6	1	7	13	4	10	1	7	13	4	10	1	7	13	4	10
M7	1	8	15	7	14	6	13	5	12	4	11	3	10	2	9

IV. MACHINE CONNECTIVITY

A set of multiphase machines, that are characterised with an odd number of phases on stator, equal to n , is under consideration. Depending on the properties of the phase number n , three different situations may arise with respect to the number of connectable machines and the individual phase numbers of connectable machines.

a) Let the number of phases n be a prime number. The number of machines that can be connected in series with phase transposition then equals

$$k = \frac{n-1}{2} \quad (22)$$

since there are $(n-1)/2$ pairs of current components that can be used for independent flux and torque control in this multiphase machine set (the remaining component, zero sequence component, will not exist since the connection of the multiphase multi-drive system is in star). All the machines are in this case of the same number of phases equal to n . The phase numbers belonging to this category are

$$n = 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, \dots \quad (23)$$

and sub-section III.B has elaborated two such phase numbers, five and seven.

b) Consider next the number of phases n that is not a prime number, but it satisfies the condition

$$n = 3^m, \quad m = 2, 3, 4, \dots \quad (24)$$

The number of machines that can be connected remains to be given with (22), i.e. $k = (n-1)/2$. However, not all k machines are in this case of the phase number equal to n . As shown in Section III.B using the nine-phase case as the example ($n = 9, m = 2$), the number of machines that can be connected is equal, according to (22) to $k = 4$. However only $k - 1$ machine are nine-phase, while the last machine is three-phase. In the case of $n = 27$ coefficient m equals $m = 3$, meaning that although a total of $k = 13$ machines can be connected in series, only certain number of these will be with 27 phases while there will be machines with nine phases and three phases as well. Actually, there will be one three-phase machine, three nine-phase machines and nine 27-phase machines. Hence for the general case of $m > 1$ the phase numbers of the machines that can be connected in series will be

$$n, \frac{n}{3}, \frac{n}{3^2}, \dots, \frac{n}{3^{m-1}} \quad (25)$$

This case can be generalised for the total number of phases n equal to

$$n = l^m, \quad m = 2, 3, 4, \dots \quad (26)$$

where l is a prime number. For example, for $n = 25, l = 5$ and $m = 2$. Hence 12 machines can be connected in series. Ten of these will be with twenty five phases while two will be five-phase. Hence the number of phases of connectable machines will be

$$n, \frac{n}{l}, \frac{n}{l^2}, \dots, \frac{n}{l^{m-1}} \quad (27)$$

Numbers of phases encompassed by this category are:

$$n = 9, 25, 27, 49, 81, 121, 125, 169, 243, \dots \quad (28)$$

It should be noted that the number of machines with smaller number of phases than n , which can be used within any multi-drive system, is limited. For example, for $n = 9, 27, 81, \dots$ there can be no more than one 3-phase machine; and no more than three 9-phase machines; etc. Similarly for $n = 25, 125, \dots$ there can be no more than two 5-phase machines. For $n = 49$ there can be no more than three 7-phase machines, for $n = 121$ there can be no more than five 11-phase machines, etc.

c) The third possible case arises when n is not a prime number and is not equal to l^m . However, n is divisible by two or more prime numbers (for example, for $n = 15$ the two prime numbers are 3 and 5). Let these prime numbers be denoted as n_1, n_2, n_3, \dots . The number of machines that can be connected is now

$$k < \frac{n-1}{2} \quad (29)$$

Ordering of machines in series connection has to follow the following rule: all the n -phase machines are at first connected in series to the source, with phase transposition. Next follow the machines with the largest prime number value out of n_1, n_2, n_3, \dots (say, n_1). This should be followed by connection of all the machines with the second largest prime number, say n_2 , etc.. This rule has to be observed, since its violation makes operation of a higher phase number machine, connected after a lower phase number machine, impossible. For example, in 15-phase case, considered in detail in section II.B, an attempt to connect at first a 5-phase machine and then a 15-phase machine means that the information regarding currents required for the operation of the 15-phase machine will be lost through current cancellation in the 5-phase machine. One thus ultimately reaches the stage where one n_1 phase machine is to be connected to a machine with n_2 phases. This is not possible since the ratio n_1/n_2 is not an integer. This is so since an attempt to connect machines of phase numbers equal to different prime numbers leads to the short circuiting of terminals.

Again, as already discussed, among these k machines only a certain number will be with n phases. The other machines that should be connectable in the multi-drive system will have phase numbers equal to n_1, n_2, n_3, \dots . However, in general, the group will be composed of machines with the phase numbers equal to n and one of the prime numbers n_1, n_2, n_3, \dots :

$$n, n_1 \quad \text{or} \quad n, n_2 \quad \text{or} \quad n, n_3 \dots \quad \text{or} \quad n, n_j \quad (30)$$

where

$$n = n_1 \cdot n_2 \cdot n_3 \cdot \dots \cdot n_j \quad (31)$$

Note that this category encompasses the situation where some of the numbers $n_1, n_2, n_3, \dots, n_j$ may be the same prime number, but there is at least one other prime number in this sequence. In other words, situation where $n = n_1 \cdot n_2 \cdot \dots \cdot n_j \cdot l^m$, $m = 2, 3, 4, \dots$ is included. The phase numbers belonging to this category are:

$$n = 15, 21, 33, 35, 39, 45, 51, 55, 57, 63, 65, 69, 75, 77, \dots \quad (32)$$

It is to be noted that categories b) and c) can be regarded as sub-categories of a more general case for which n is not a prime number. They have in common that two or more phase numbers appear in the series connection with phase transposition of the multiphase machines. However, the total number of connectable machines is not the same (it is given with (22) and (29), respectively).

A summary of all possible situations that can arise with $n = \text{odd number}$ is given in Table VI.

V. VECTOR CONTROL OF A SEVEN-PHASE THREE-MOTOR DRIVE SYSTEM

A. Indirect vector controller for a seven-phase induction machine

A standard indirect method of achieving rotor flux oriented control of a current-fed induction machine is considered here. The basic form of the vector controller remains to be the same as for a three-phase machine and the only difference is in the co-ordinate transformation, where seven phase current references are generated by means of co-ordinate transformation described with (11) and (15), instead of three. The indirect vector controller for operation in the base speed (constant rotor flux) region is illustrated in Fig. 5 and is of the same form for all

Table VI: Possible situations with phase number $n =$ an odd number.

$n =$ an odd number, ≥ 5			
		Number of connectable machines	Number of phases of machines in the multi-drive system
$n =$ a prime number	$n = 5, 7, 11, 13, \dots$	$k = \frac{n-1}{2}$	n
$n \neq$ a prime number	$n = l^m, \quad m = 2, 3, 4, \dots$	$k = \frac{n-1}{2}$	$n, \frac{n}{l}, \frac{n}{l^2}, \dots, \frac{n}{l^{m-1}}$
	$n = n1 \cdot n2 \cdot n3 \cdot \dots \cdot nj$	$k < \frac{n-1}{2}$	$n, n1$ or $n, n2$ or $n, n3, \dots$ or n, nj
	$n = n1 \cdot n2 \cdot \dots \cdot nj \cdot l^m, \quad m = 2, 3, 4, \dots$	$k < \frac{n-1}{2}$	$n, n1$ or $n, n2, \dots$ or n, nj or $\frac{n}{l}, \frac{n}{l^2}, \dots, \frac{n}{l^{m-1}}$

three machines. It should be noted that the three machines can be of different types (i.e. the group may consist of induction motors, permanent magnet synchronous motors and synchronous reluctance motors) and they can be of different ratings as well. The drive system considered here is taken as consisting of three identical seven-phase induction machines, for the sake of simplicity. All the relevant data of the machine are given in the Appendix.

B. Creation of inverter current references

Phase current references for the three machines are, according to Fig. 5, determined with ($\alpha = 2\pi/7$):

$$\begin{aligned}
 i_{a(j)}^* &= \sqrt{\frac{2}{7}} [i_{ds(j)}^* \cos \phi_{r(j)} - i_{qs(j)}^* \sin \phi_{r(j)}] \\
 i_{b(j)}^* &= \sqrt{\frac{2}{7}} [i_{ds(j)}^* \cos(\phi_{r(j)} - \alpha) - i_{qs(j)}^* \sin(\phi_{r(j)} - \alpha)] \\
 i_{c(j)}^* &= \sqrt{\frac{2}{7}} [i_{ds(j)}^* \cos(\phi_{r(j)} - 2\alpha) - i_{qs(j)}^* \sin(\phi_{r(j)} - 2\alpha)] \\
 i_{d(j)}^* &= \sqrt{\frac{2}{7}} [i_{ds(j)}^* \cos(\phi_{r(j)} - 3\alpha) - i_{qs(j)}^* \sin(\phi_{r(j)} - 3\alpha)] \\
 i_{e(j)}^* &= \sqrt{\frac{2}{7}} [i_{ds(j)}^* \cos(\phi_{r(j)} + 3\alpha) - i_{qs(j)}^* \sin(\phi_{r(j)} + 3\alpha)] \\
 i_{f(j)}^* &= \sqrt{\frac{2}{7}} [i_{ds(j)}^* \cos(\phi_{r(j)} + 2\alpha) - i_{qs(j)}^* \sin(\phi_{r(j)} + 2\alpha)] \\
 i_{g(j)}^* &= \sqrt{\frac{2}{7}} [i_{ds(j)}^* \cos(\phi_{r(j)} + \alpha) - i_{qs(j)}^* \sin(\phi_{r(j)} + \alpha)]
 \end{aligned} \tag{33}$$

Subscript (j) in (33) denotes the machine under consideration, i.e. ($j = 1, 2, 3$).

Inverter phase current references are built next, respecting the connection diagram of Fig. 3:

$$\begin{aligned}
 i_A^* &= i_{a1}^* + i_{a2}^* + i_{a3}^* & i_B^* &= i_{b1}^* + i_{c2}^* + i_{d3}^* \\
 i_C^* &= i_{c1}^* + i_{e2}^* + i_{g3}^* & i_D^* &= i_{d1}^* + i_{g2}^* + i_{c3}^* \\
 i_E^* &= i_{e1}^* + i_{b2}^* + i_{f3}^* & i_F^* &= i_{f1}^* + i_{d2}^* + i_{b3}^* \\
 i_G^* &= i_{g1}^* + i_{f2}^* + i_{e3}^*
 \end{aligned} \tag{34}$$

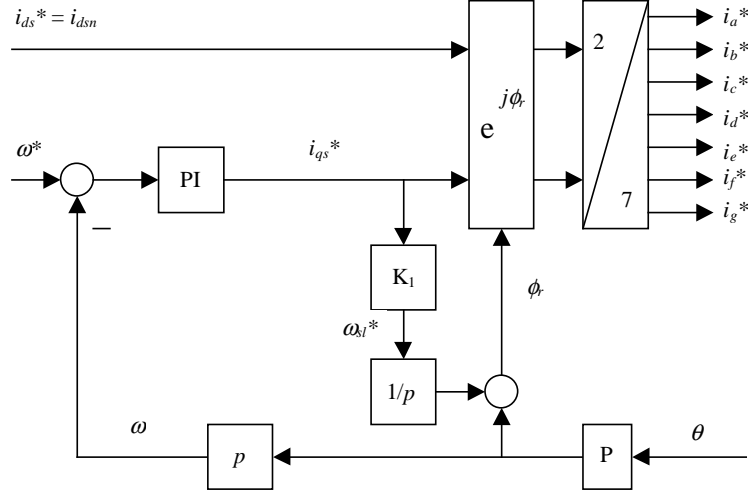


Fig. 5: Indirect vector controller for a seven-phase induction machine ($p = \text{Laplace operator}$, $K_1 = 1/(T_r i_{ds}^*)$).

Current control is performed in the stationary reference frame, using either hysteresis current control or ramp comparison control. It is assumed for the simulation purposes in the next Section that the current controlled VSI can be treated as an ideal current source, so that inverter phase current references of (34) are taken as equal to the inverter output phase currents.

C. Source voltages

Inverter phase voltages are governed by the connection diagram of Fig. 3. They are determined with an appropriate summation of stator phase voltages of individual machines, respecting the phase transposition. Hence

$$\begin{aligned}
 v_A &= v_{as1} + v_{as2} + v_{as3} & v_B &= v_{bs1} + v_{cs2} + v_{ds3} & v_C &= v_{cs1} + v_{es2} + v_{gs3} \\
 v_D &= v_{ds1} + v_{gs2} + v_{cs3} & v_E &= v_{es1} + v_{bs2} + v_{fs3} & v_F &= v_{fs1} + v_{ds2} + v_{bs3} \\
 v_G &= v_{gs1} + v_{fs2} + v_{es3}
 \end{aligned} \tag{35}$$

VI. VERIFICATION OF THE CONCEPT

The proposed configuration of a multiphase multi-motor drive system is at first verified by simulating a seven-phase three-motor drive system. Both torque mode and speed mode of operation are elaborated and the results are reported in the following two sub-sections. The description of the experimental rig and some experimental results, which further confirm the validity of the concept, are provided in the third sub-section.

A. Simulation verification - Torque mode of operation

As already noted, the inverter is treated as an ideal current source. The three induction machines are at this stage represented with the phase variable model (1)-(9). The machine models obtainable using general theory of electrical machines are therefore not utilised and such an approach leads to an ultimate proof of the concept. Since stator currents are known for each of the three machines (they are determined with source currents of (34) and the phase transposition of Fig. 3), stator current derivatives are calculated and stator phase voltages of (1) are then reconstructed in order to find the source voltages of (35).

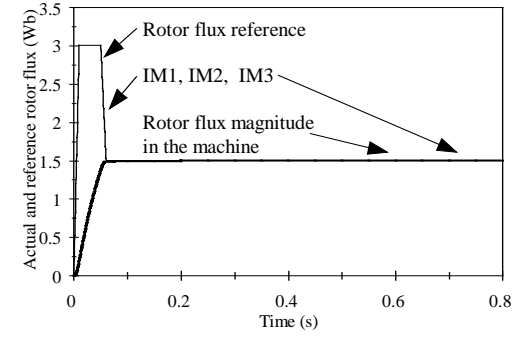
A forced excitation is at first initiated for all the three machines in the following way. Stator d-axis reference is ramped to twice the rated value (i.e. 7.16 A) in the interval from $t = 0$ to $t = 0.01$ s. It is then kept at this value until 0.05s and is finally ramped back to rated (i.e. 3.58 A) from 0.05 to 0.06s. Once when the rotor flux is established in the machines, a torque reference of different value is applied to the three machines in different instants of time, in a ramp-wise manner. Torque reference is later on brought back to zero for all the three machines, again in the ramp-wise manner. For the first machine the torque reference is ramped from zero to 1.333 rated torque (i.e. 15.56 Nm) from 0.3 s to 0.31 s and is reduced back to zero from 0.55 s to 0.56 s. For the second machine the torque reference is rated (i.e. 11.667 Nm) and the two time intervals are 0.35 to 0.36 s and 0.5 to 0.51 s. Torque reference of the third machine equals 0.6667 times rated torque (i.e. 7.778 Nm) and the corresponding time intervals are 0.4 to 0.41 s and 0.65 to 0.66 s. The first set of results is shown in Fig. 6.

As can be seen from Fig. 6a, forced excitation establishes required rotor flux value in all the three machines without any undesired coupling between the machines. Rotor flux in the machine attains the reference value, confirming the absence of any x-y rotor flux components. Rotor flux remains completely undisturbed during the subsequent torque transients. The result in Fig. 6b illustrates rotor phase 'a' flux for the first two machines and confirms once more that rotor flux is not affected by the existence of the stator current x-y components. Torque reference and actual torque are shown in Fig. 6c for all the three machines. Due to the assumed ideal current feeding torque reference and actual torque coincide for each of the three machines. Torque developed by any of the three machines is not affected at all by the torque produced by the two other machines, confirming that the control of the three machines is completely decoupled through the introduced phase transposition. Speed responses in Fig. 6d are therefore smooth and the same as would have been obtained with just one vector controlled seven-phase machine supplied from a seven-phase source. Stator current references for phase 'a' of each of the three machines, generated according to (33), are shown in Fig. 6e. These are of course sinusoidal in any steady state operation of a running machine. The impact of the series connection of stator windings is evidenced in the display of stator phase 'a' voltages of the three machines, shown in Fig. 6f. These are highly distorted due to the flow of flux/torque producing stator currents of all the three machines through all the phases of the three machines. Due to the phase transposition x-y components appear in stator voltages, caused by the flow of x-y stator current components.

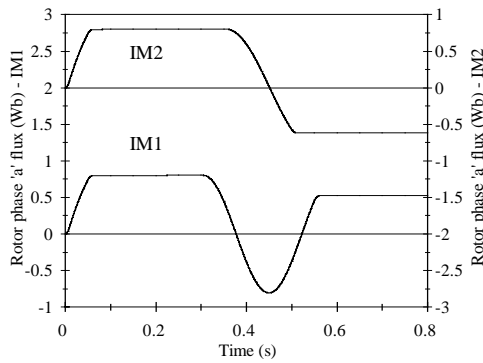
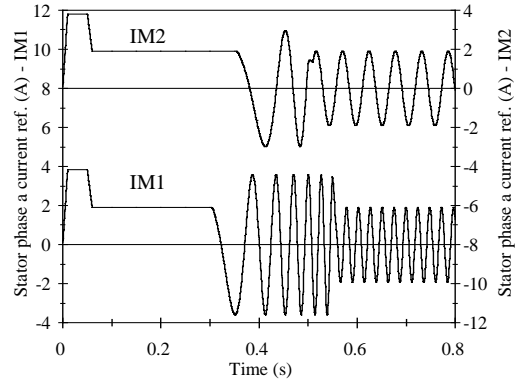
Inverter output currents and inverter output phase voltages, obtained using (34) and (35), respectively, are shown in Figs. 7 and 8, respectively, for the first six phases of the seven-phase system. The consequence of the series connection of the three machines is more than evident in these waveforms. However, a current controlled voltage source inverter can generate any required waveform of a current reference and therefore the non-sinusoidal waveform of the inverter current references will not present any difficulty in an actual implementation. This statement is fully verified by experimental results presented in sub-section C.

B. Simulation verification - Speed mode of operation

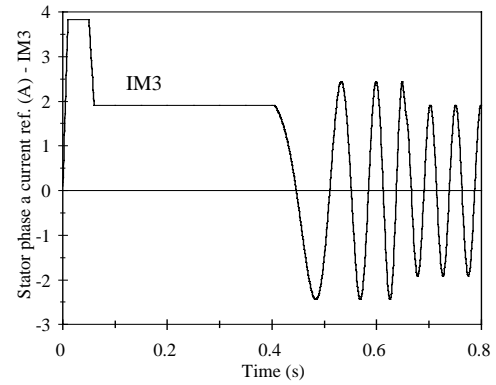
Speed mode of operation is investigated next. The machines are now represented with the d-q models in the stationary reference frame. A mixture of an acceleration transient, a reversing transient and a load rejection transient is investigated. Torque limit is set to twice the rated torque for all the three machines (23.33 Nm). All the machines are pre-excited to the rated rotor flux, in the manner illustrated already in Fig. 6. Machine 1 is at standstill, machine 2 runs at rated speed (1428 rpm) and machine 3 runs at 2/3 of the rated speed (952 rpm) (all



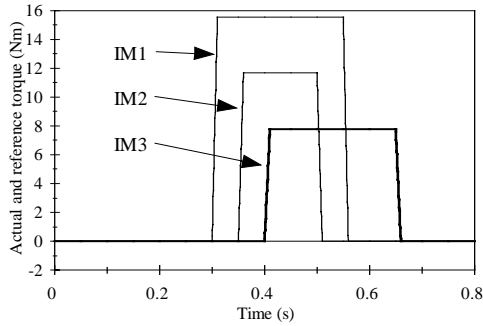
a.



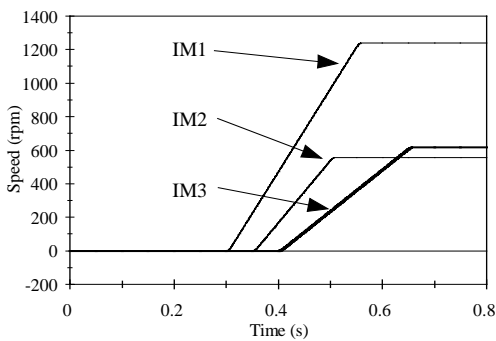
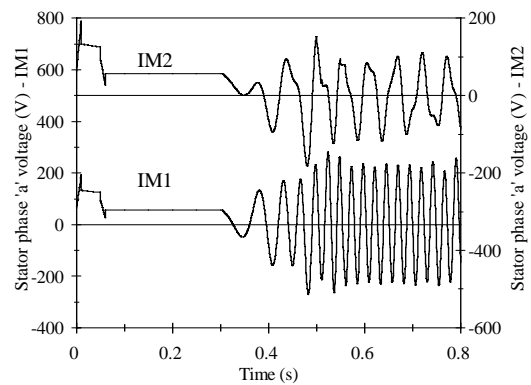
b.



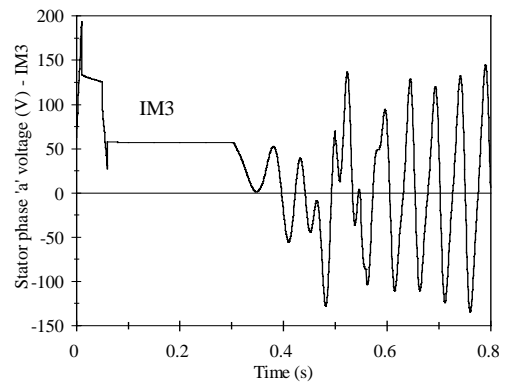
e.



c.



d.



f.

Fig. 6: Torque mode of operation of the seven-phase three-motor drive system: a. Rotor flux reference and rotor flux magnitude in the machines; b. Rotor phase 'a' flux in the first two machines; c. Torque reference and actual torque for the three machines; d. Speed response for the three machines; e. Stator phase 'a' current references; f. Stator phase 'a' phase voltages.

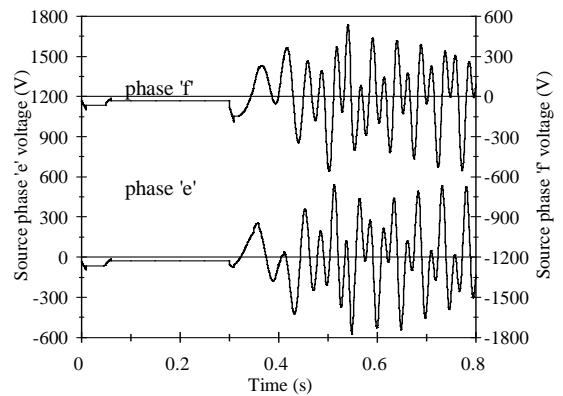
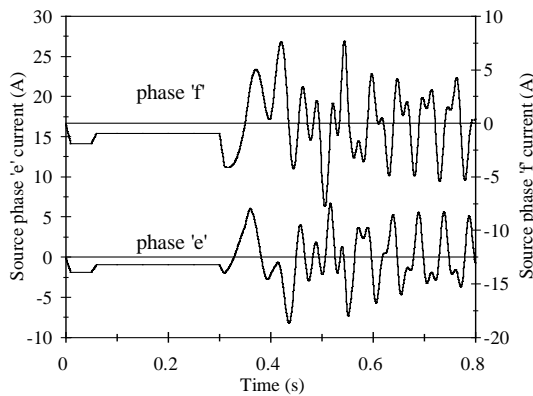
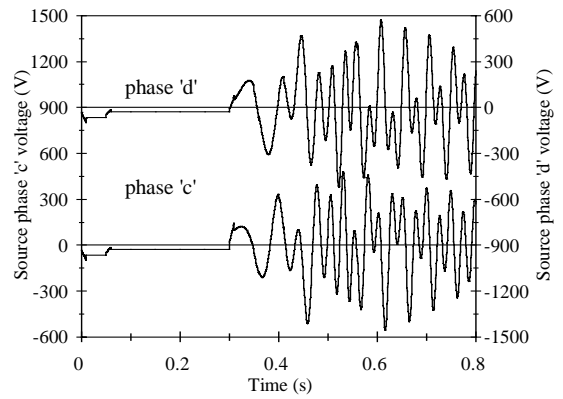
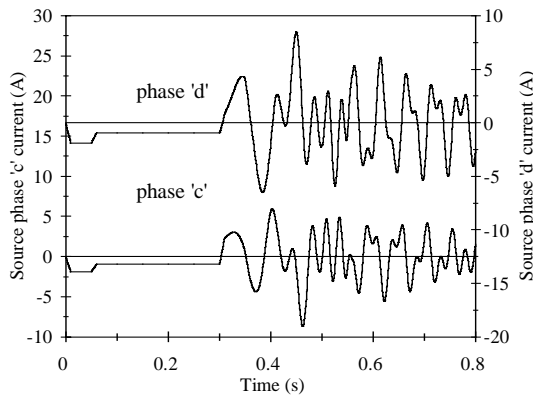
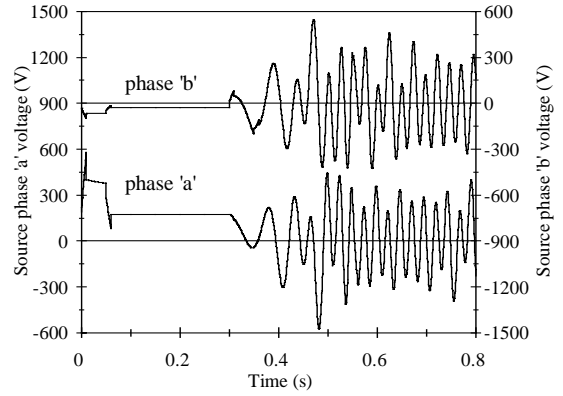
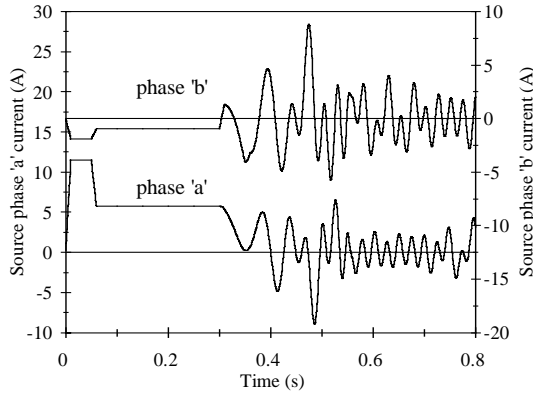


Fig. 7: Inverter output currents for the first six phases.

Fig. 8: Inverter output phase voltages for the first six phases.

three under no-load conditions). Rated speed command is applied to IM1 at $t = 1.1$ s in a ramp wise manner (0.1 s ramp). A speed command of negative rated value is applied to IM2 at $t = 1$ s in a ramp wise manner (ramp of 0.2s). A load of 60% of the rated (7 Nm) is applied to IM3 at $t = 1.3$ s in a step wise manner. Figure 9 shows the results. Rotor flux (Fig. 9a) remains completely undisturbed regardless of what takes place in any of the three machines. Torque response (Fig. 9a) indicates complete decoupling of the control of the three machines, leading to the fastest possible speed response (Fig. 9b). Stator phase 'a' current references, created by the vector controller of Fig. 5 for each of the three machines, are shown in Fig. 9b. The impact of the series connection of the stator winding phases is evident from the distorted stator phase 'a' voltages in Fig. 9c. Source voltages and source currents are finally given in Fig. 10 for the first two phases of the seven-phase system. In principal, the same observations apply to the source variables as for the torque mode of operation.

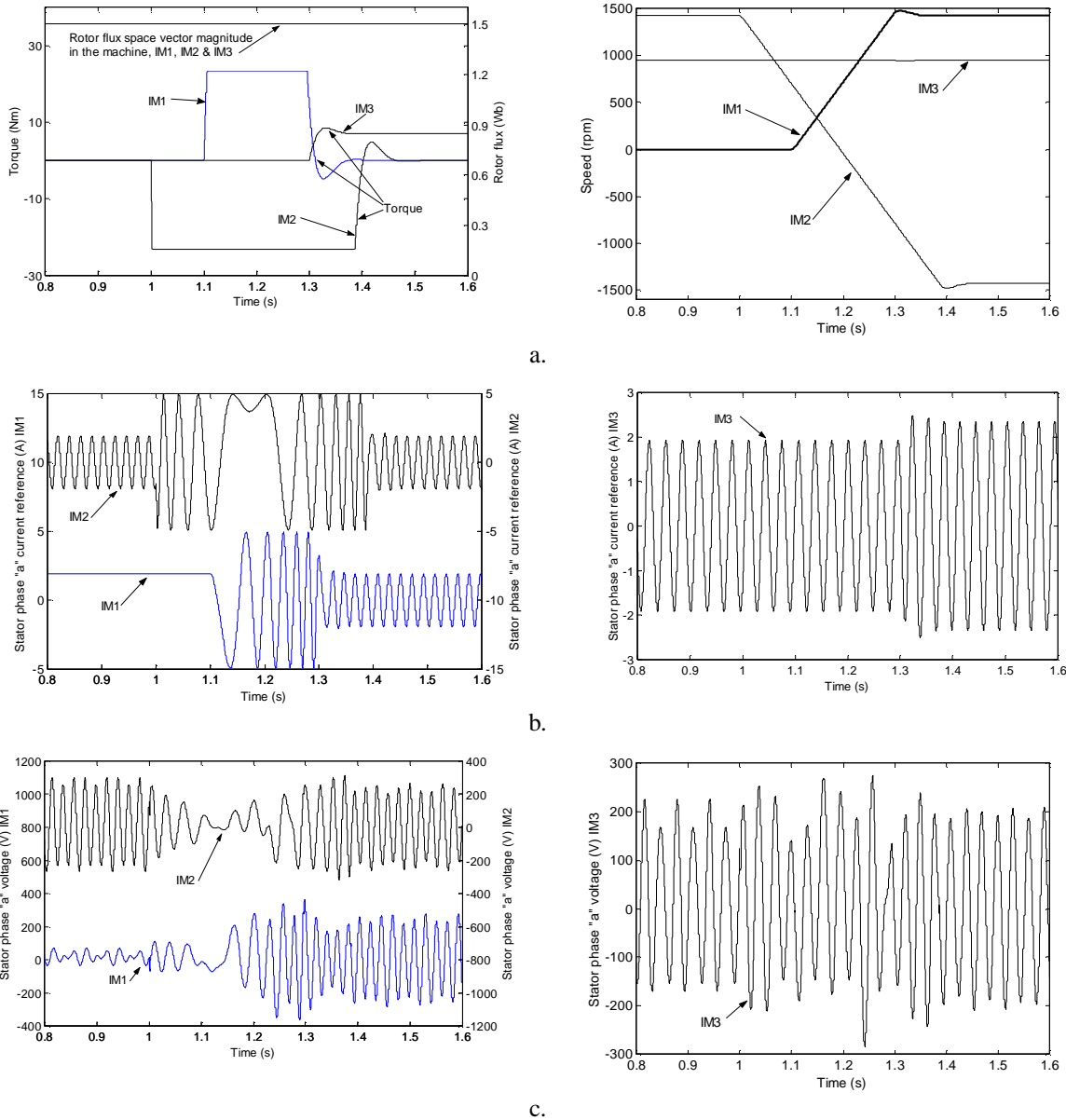


Fig. 9: A mixture of an acceleration transient (IM1), a reversing transient (IM2) and a load disturbance rejection (IM3) in the speed mode of operation of a vector-controlled seven-phase three-motor drive system:

- Rotor flux, torque and speed response of the three machines.
- Stator phase 'a' current references.
- Stator phase 'a' phase voltages.

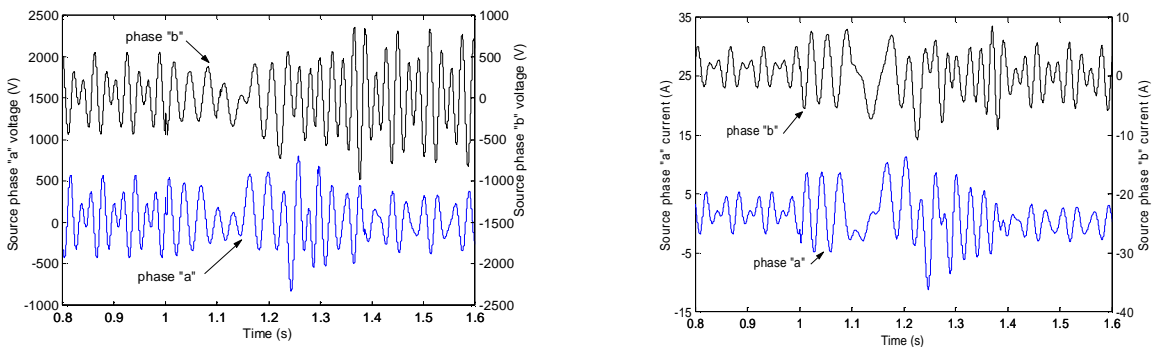


Fig. 10: Inverter output phase voltages and currents for the first two phases, for the mixture of transients illustrated in Fig. 9.

C. Experimental verification – The experimental rig and preliminary experimental results

The experimental rig (Fig. 11) is designed on the basis of four commercially available three-phase inverters. The mains inputs of the diode bridge rectifiers and the DC links are paralleled and the rig can therefore be used for any supply phase number from five to twelve. Each of the inverters is equipped with a DSP. All the inverter currents are measured using LEM sensors and DSPs within inverter units perform closed loop current control in the stationary reference frame, using ramp-comparison method. The switching frequency is 10 kHz. Each of the four three-phase inverter units has one resolver/encoder input. The rig can therefore be used to control at most four series-connected motors in sensed mode (corresponding to supply phase numbers equal to nine and ten). If at least one machine operates in speed sensorless mode, it is possible to use the rig for control of up to five series-connected machines (corresponding to the supply phase number equal to eleven). Table VII summarises the intended capabilities of the rig, on the basis of the utilised supply phase number.

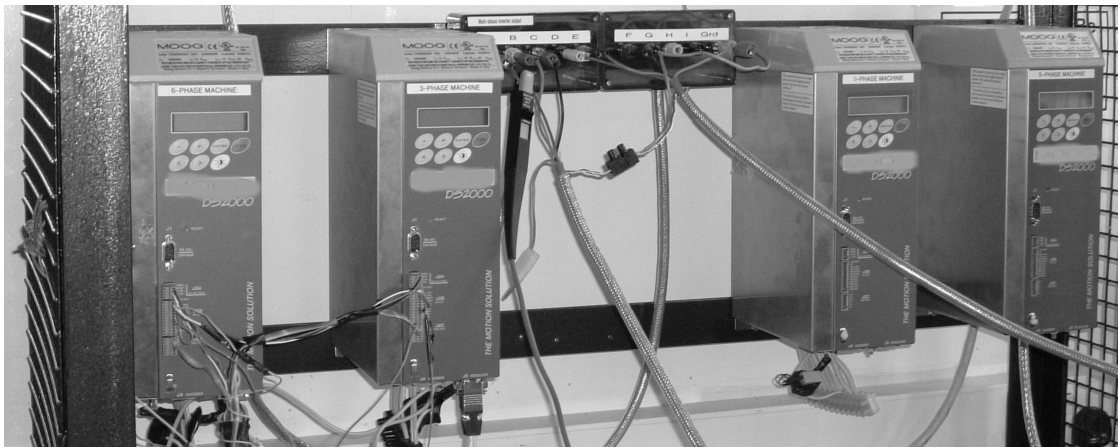


Fig. 11: Illustration of the twelve-phase experimental rig, capable of driving up to four series-connected motors in sensed mode (up to five if at least one machine operates in speed sensorless mode).

Table VII: Experimental rig capability, as function of the utilised number of inverter phases.

	Utilised number of the inverter phases (maximum = 12)							
	5	6*	7	8*	9	10*	11***	12**
Number of connectable machines	2	2	3	3	4	4	5***	4**

* Even supply system phase numbers are elaborated in [24].

** The twelve-phase supply can control at most four series-connected motors, rather than five (see [24]).

*** At least one machine must operate in speed sensorless mode.

Although the rig could be used for control of up to four (five) series-connected machines, the intention is currently to verify the concepts developed here and in [24] by experimentally investigating the operation of the two two-motor drive systems listed in Table VII (the five-phase and the six-phase case).

The control code is written in C and it performs closed loop speed control utilising indirect rotor flux oriented controllers of the form shown in Fig. 5 and calculations analogous to those of (33) and (34). The inverter current references are passed to the DSPs from a PC, through a dedicated interface card. The rig is currently in the commissioning stage and some preliminary steady state results are reported here. Figure 12

illustrates the measured instantaneous inverter phase current in the series-connected two-motor drive system. The reference speed commands for the two induction machines are 50 Hz and 16.67 Hz (electrical), respectively, and both machines operate under no-load conditions. A distortion of the inverter current, similar to the one observed in simulation results in Figs. 7 and 9 during intervals of steady state operation, is evident. The only principal difference between the simulation results and the current in Fig. 12 is that the seven-phase three-motor case was simulated, so that the steady state current was composed of three sinusoidal components of different frequencies. In the two-motor case of Fig. 12 the inverter current is composed of two sinusoidal (neglecting the PWM ripple) current components, corresponding to the operating frequencies of the two machines. This is confirmed in Fig. 13, where the harmonic spectrum (low frequency part) of the inverter current is shown. The presence of two fundamental harmonics (of RMS values governed with the reference stator d-axis currents), corresponding to the operating speeds of 16.67 Hz and 50 Hz, is evident.

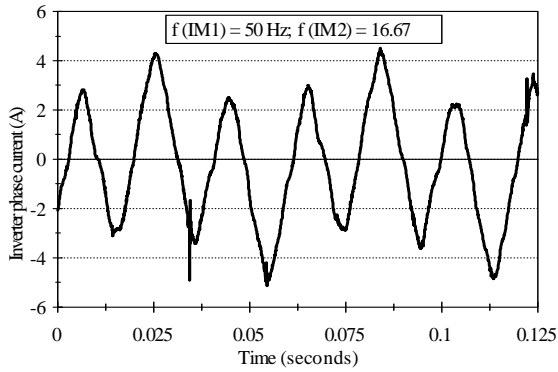


Fig. 12: Measured inverter phase current in the series-connected two-motor drive system.

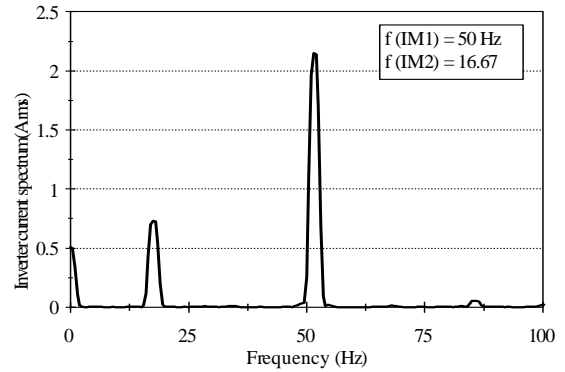


Fig. 13: Low frequency part of the harmonic spectrum of the inverter current shown in Fig. 12.

VII. DISCUSSION

The concept of the multiphase multi-motor drive system, developed in this paper, offers three main advantages when compared to an equivalent three-phase multi-motor system with common DC link and a separate three-phase VSI for each machine. The first one is the potential for saving in the number of inverter legs. For example, a five-phase two-motor system requires five inverter legs, compared to six inverter legs needed by an equivalent two-motor three phase system. A seven phase three-motor system requires only seven legs, compared to nine required by the equivalent three-phase three-motor system. The saving in the number of legs increases as the number of phases increases. Leaving aside the question of what a practical limit on the number of phases is, a twenty nine phase system would need 29 inverter legs for supply of fourteen motors, while an equivalent three-phase system would require 42 inverter legs (a saving of 13 inverter legs). Saving in the number of inverter legs translates into application of a smaller number of semiconductors and associated components (drivers, protection circuits, etc.). Due to the smaller overall count of components, a multiphase multi-motor drive system is expected to have a better reliability than the equivalent three-phase system. Some improvement in the inverter efficiency is anticipated as well.

The second advantage of a multiphase multi-motor drive system is in the easiness of the implementation of the vector control algorithm for all the machines of the group within a single DSP. In the version discussed in

detail for the seven-phase case, the DSP would perform three indirect rotor flux oriented control algorithms in parallel, giving at the output the references for the inverter phase currents. In the experimental two-motor drive systems two vector control algorithms are executed in parallel and current control is implemented using the ramp comparison current control technique.

The third advantage of the multi-phase series-connected drive systems is the possibility of direct utilisation of the braking energy, developed by some of the machines of the group, by the other machines that are operating in motoring. This means that the braking energy does not have to be returned back to the DC link and therefore it does not circulate through the inverter. In other words, the system provides full regenerative braking as long as the total braking energy, developed by some of the machines in the system, is smaller than the total motoring energy, required by the other machines in the system. Only once when the braking energy exceeds the required motoring energy, some power is returned to the DC link and dynamic braking (or some other type of braking) becomes necessary.

An obvious drawback of the concept is an increase in the stator winding losses (and a considerably smaller increase in the stator iron losses) due to the flow of the flux/torque producing currents of all the machines through stator windings of all the machines (note that rotor winding losses are not affected). This will inevitably decrease the efficiency of every individual machine in the multi-motor system and will yield an overall reduction in the total efficiency of the drive system, when compared to an equivalent three-phase drive system.

The true potential of the proposed multi-motor drive systems for industrial applications cannot be assessed at present in any credible manner, since neither multi-phase machines nor multi-phase inverters are available on the market. This renders any attempt to perform a viable economic analysis impossible. The authors nevertheless hope that at least some of the multi-phase multi-motor drive systems, obtainable on the basis of the concept developed in the paper, may prove as a worthy solution for certain applications in near future.

VIII. CONCLUSION

The paper develops a novel concept for a multi-motor drive system, which enables independent control of a set of AC machines supplied from a single current-controlled voltage source inverter. The number of connectable machines depends on the number of phases of the stator winding and on the properties of the phase number. The stator multiphase windings have to be connected in series with an appropriate phase transposition, in order to achieve the independent control of the machines in the system. The concept is developed in a systematic manner, using general theory of electrical machines, and is valid regardless of the type of the AC machine, so that different machine types can be used within the same multi-motor drive system. The necessary phase transposition in the winding connections is established by analysing the properties of the decoupling transformation matrix and so-called connectivity matrix is formed for an arbitrary number of phases. The analysis in the paper is restricted to an odd number of phases, since any even number of phases requires one more inverter leg than the previous odd number, while enabling series connection of at best the same number of machines.

On the basis of the developed general scheme for the series connection of the multiphase windings, a couple of specific phase numbers are analysed further and corresponding connectivity matrices and connection diagrams are presented. The cases covered are the five-phase, seven-phase, nine-phase and fifteen-phase systems. A classification of all the possible situations with regard to the number of phases is performed next and it is shown

that the highest number of connectable machines result when the number of phases is either a prime number or a prime number at a certain power.

The concept is at first verified by simulation of a three-motor seven-phase drive system. Both torque mode and speed mode of operation are examined and it is shown that completely decoupled and independent vector control of the three machines is possible with the proposed series connection. Preliminary experimental results, related to the steady state operation of a series-connected two-motor drive system, are further provided to confirm the feasibility of the series-connected drive structures. Last but not least, the main advantages and shortcomings of the series-connected multi-phase multi-motor drives, when compared to their three-phase counterparts, are highlighted.

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ACKNOWLEDGEMENT

The authors gratefully acknowledge support provided for the work on this project by the EPSRC, under the standard research grant number GR/R64452/01, and by Semikron Ltd. Mr. M. Jones acknowledges financial support provided for his PhD studies by the IEE, through the IEE Robinson Research Scholarship.

APPENDIX

Per-phase equivalent circuit parameters of the 50 Hz seven-phase induction motor used in simulations:

$$R_s = 10 \Omega \quad R_r = 6.3 \Omega \quad L_{ls} = L_{lr} = 0.04 \text{ H} \quad L_m = 0.42 \text{ H}$$

Inertia and number of pole pairs: $J = 0.03 \text{ kgm}^2$, $P = 2$.